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Numerical Modeling on Mechanical Properties of Cemented Phosphogypsum Stabilized Soil

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Abstract

Phosphogypsum is an industrial waste with a large stock and will show a great threat to people's lives. The study of its mechanical properties is particularly important for engineering applications. The objective of the work is to discuss the influence mechanism of cemented phosphogypsum-stabilized soil by different parameters and provide a research basis for the engineering application of phosphogypsum as road subgrade. The main analysis methods are as follows. The published mechanical test data of cemented phosphogypsum-stabilized soil are firstly collected in this work, and the numerical models for describing the compaction properties, liquid-plastic limit properties, unconfined compressive strength, and the cracking properties of cemented phosphogypsum-stabilized soil are then established by numerical fitting. Based on the verified model, the effects of different parameter factors on the mechanical behavior of cemented phosphogypsum-stabilized soil are finally carried out. The results show that the numerical model can effectively predict the influence of different factors on the mechanical properties of materials and is in good agreement with the test results. The novelty of this work is establishing the numerical modeling on the mechanical properties of cemented phosphogypsum-stabilized soil, considering the effects of different parameter factors.

Keywords: Phosphogypsum; Mechanical Properties; Numerical Modeling; Stabilized Soil.

1. Introduction

Phosphogypsum, as one of the industrial wastes, can be produced by chemical enterprises in the process of decomposing phosphate rock and extracting phosphoric acid with sulfuric acid [1]. Due to the existing insufficiently reacted sulfuric acid and other hazardous substances, it has shown a great threat to people's production, life, and health [2-5]. It is urgent to accelerate the comprehensive utilization of phosphogypsum, especially its application in road engineering [6]. If the recycled phosphogypsum can be used as a road base material, it can not only turn waste into treasure and promote the conservation and intensive use of resources but also produce better economic and ecological benefits [7, 8].

In recent years, experts and scholars have carried out a lot of research on the engineering application of phosphogypsum, including cement phosphogypsum and lime phosphogypsum [9]. The phosphogypsum, treated as a waste by-product, its rational use will certainly provide a relatively cheap building material market for the construction industry [9]. In order to better apply the phosphogypsum to engineering practice, its mechanical properties need

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extensive and in-depth research. The unconfined compressive strength and other mechanical properties of phosphogypsum under different cement contents were studied by Wan et al. [10]; although some valuable conclusions are obtained in such work, the results for other cases with different contents or different parameters, however, are not fully given. Zeng reported the effect on the strength and microstructure of cement-phosphogypsum-stabilized soils by the wetting-drying [11]; the properties such as compaction properties, liquid-plastic limit properties, cracking properties, unconfined compressive strength, and water stability properties were not discussed yet. The mechanism for the phosphogypsum to improve the strength of silt was studied by Wang et al. [12], while the question of how to improve or affect the above-mentioned mechanical behaviors of phosphogypsum-stabilized soil has not been widely discussed, especially in the case of cement phosphogypsum. Zheng et al. [13] discussed the influence of different contents of phosphogypsum on the stability of phosphogypsum soil by experimental test, and due to the hard test process, it usually has some shortcomings, such as less test data and difficulty mastering the effect law. The machine learning theory is used to analyze and predict the unconfined compressive strength of phosphogypsum stabilized soil by Min et al. [14], however, a large number of experimental data are needed as data sets in the research process, which makes the research process more complicated. The water stability analysis of phosphogypsum as subgrade filling with different content and different proportions of materials is discussed by Wu et al. [15]. Although some valuable conclusions have been obtained about the water stability of phosphogypsum as subgrade filling material, its water stability has not been fully given with the change of material parameters.

According to the above analysis, most of the published research is mainly focused on the experimental testing of the mechanical properties of cement phosphogypsum materials; the numerical modeling of the mechanical properties of cemented phosphogypsum-stabilized soils, however, was rarely reported. Due to the lack of the numerical model, when one wants to discuss the relationship between the mechanical properties and the changing parameter values, it is always showing the drawbacks of incomplete experimental data or needing more experimental test data to fully realize the predictive material mechanical properties. For obtaining more mechanical properties of such materials with the changing material parameters and ratios, it is very necessary to establish a numerical analysis model for the cemented phosphogypsum-stabilized soils to express its detailed mechanical properties with the changing material parameters and ratios.

In this work, the published test data of the cement phosphogypsum-stabilized soils are firstly sorted out, and a mathematical fitting model to express the relationship between the mechanical properties and the corresponding influencing factors is then established based on the existing test data. And the workflow chart of the whole work can be seen in Figure 1. The establishment of the model in this work can be used to accurately predict the influence of different factors such as material parameters and ratio on the mechanical properties of cement phosphogypsum, and it also can effectively reduce the number of the experimental tests. This work shall provide theoretical support for the further engineering application of cement phosphogypsum.

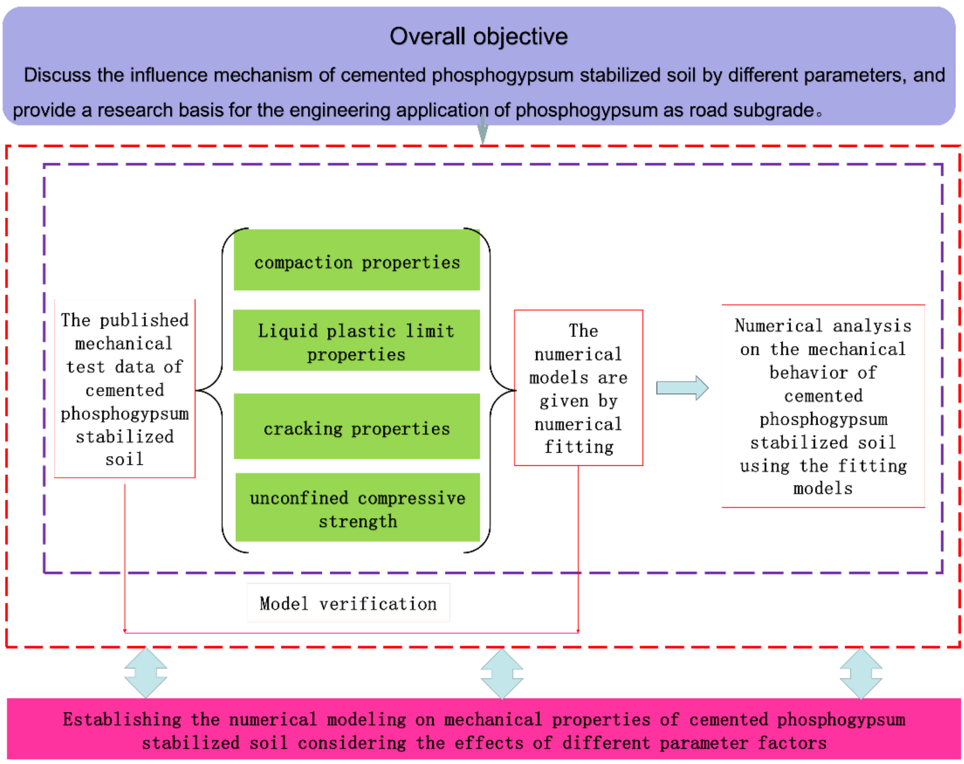


Figure 1. Flowchart of the research methodology

The remainder of the paper is organized as follows. The experimental data of mechanical properties for the cement phosphogypsum-stabilized soil are briefly reviewed in Section 2. Section 3 gives the numerical modeling of mechanical properties for the cement phosphogypsum-stabilized soil. The validation of the model is considered in Section 4. Section 5 is devoted to simulating the numerical prediction of mechanical properties of cemented phosphogypsum-stabilized soil. Finally, some concluding remarks are given in Section 6.

2. Experimental Data of Mechanical Properties for the Cement Phosphogypsum Stabilized Soil

2.1. The Compaction Characteristics and the Liquid-Plastic Limit Properties

Zhang et al. [16] used the mix compaction test to investigate the compaction characteristics of cement phosphogypsum-stabilized soil, and the compaction test data of the optimum moisture content and maximum dry density for cemented phosphogypsum-stabilized soil with the variation of the cement content, phosphogypsum content, and red clay content were given in Table 1. Chen & Dai [17] carried out an experimental test study on the plastic-liquid limit characteristics of cemented phosphogypsum-stabilized soils; the relationship between the plastic limit, liquid limit, and plasticity index with the changing cement contents and phosphogypsum contents for cemented phosphogypsum-stabilized soil was given in Table 2.

Table 1. Compaction characteristics data of the cement phosphogypsum stabilized soil [16]

Cement (%)	Phosphogypsum (%)	Red clay (%)	Optimum Moisture Content (%)	Maximum Dry Density (g/cm^3)
3	3	94	30.3	1.498
3	6	91	29.18	1.502
3	9	88	27.83	1.54
3	24.3	72.7	26.96	1.45
3	32.3	64.7	25.84	1.505
3	48.5	48.5	25.47	1.519
5	5	90	26.15	1.513
5	10	85	27.41	1.518
5	15	80	26.35	1.540
5	23.8	71.5	27.11	1.463
5	31.7	63.3	25.92	1.538
5	47.5	47.5	22.88	1.546
7	7	86	27.2	1.491
7	14	79	27.98	1.523
7	21	72	27.32	1.522
7	23.3	46.7	25.21	1.527
7	31	62	23.93	1.544
7	46.5	46.5	24	1.512

Table 2. Test data of the plastic-liquid limit for cemented phosphogypsum stabilized soil [17]

Cement (%)	Phosphogypsum (%)	Plastic Limit	Liquid Limit	Plasticity Index
3	3	49.9	80.1	30.2
3	6	49.3	78.6	29.3
3	9	48.2	77.8	29.6
3	24.3	41.2	70.1	28.9
3	32.3	39	66.2	27.2
3	48.5	31	55.5	24.5
5	5	49.4	79.5	30.1
5	10	49.1	75.2	26.1
5	15	48.7	73.2	24.5
5	23.8	41.2	67.3	26.1
5	31.7	36.5	62.2	25.7
5	47.5	29.6	52.3	22.6
7	7	48.8	78.3	29.5
7	14	48.1	75.2	27.1
7	21	45.6	72.1	26.5
7	23.3	43.6	70.2	26.6
7	31	40.8	66.4	25.6
7	46.5	29.4	53.9	24.5

2.2. The Unconfined Tensile Strength

Li et al. [18] conducted the experimental tests on cement phosphogypsum unconfined compressive strength, and the relationship between the unconfined compressive strength and the cement contents, phosphogypsum contents, and the red clay contents for cemented phosphogypsum stabilized soil can be shown in Table 3.

Table 3. Experimental data of unconfined compressive strength for cement phosphogypsum stabilized soil [18]

Cement (%)	Phosphogypsum (%)	Red Clay (%)	Time (Days)	Unconfined Compressive Strength (MPa)
4	4	92	7	0.81
4	8	88	7	0.68
4	12	84	7	0.75
6	6	88	7	1.21
6	12	82	7	1.03
6	18	76	7	1.28
8	8	84	7	0.98
8	16	76	7	1.32
8	24	68	7	1.24
10	10	80	7	1.15
10	20	70	7	1.20
10	30	60	7	2.08
4	4	92	14	1.32
4	8	88	14	1.13
4	12	84	14	1.24
6	6	88	14	1.75
6	12	82	14	1.53
6	18	76	14	1.82
8	8	84	14	1.48
8	16	76	14	1.84
8	24	68	14	1.81
10	10	80	14	1.64
10	20	70	14	1.79
10	30	60	14	2.42
4	4	92	28	1.51
4	8	88	28	1.34
4	12	84	28	1.43
6	6	88	28	1.92
6	12	82	28	1.76
6	18	76	28	2.08
8	8	84	28	1.64
8	16	76	28	2.16
8	24	68	28	2.05
10	10	80	28	1.81
10	20	70	28	1.95
10	30	60	28	2.71

2.3. The Fissure Rate Characteristics

Li et al. [18] also published the experimental data on the fissure rate characteristics of cemented phosphogypsum-stabilized soils, and the relationship between the fissure rate and the cement contents, the phosphogypsum contents, and the red clay contents for both the highly and the lowly doped phosphogypsum were shown in Table 4 and Table 5, respectively.

Table 4. The fissure rate test data for high doped phosphogypsum [18]

Cement (%)	Phosphogypsum content: red clay content	Fissure rate (%)
3	1:1	0.64
3	1:2	0.81
3	1:3	1.11
5	1:1	0
5	1:2	0.74
5	1:3	1.45
7	1:1	0
7	1:2	0.49
7	1:3	1.24

Table 5. The fissure rate for low doped phosphogypsum [18]

Cement (%)	Cement content : Phosphogypsum content	Fissure rate (%)
3	1:1	2.84
3	1:2	1.34
3	1:3	1.17
5	1:1	2.77
5	1:2	1.31
5	1:3	1.13
7	1:1	2.33
7	1:2	1.24
7	1:3	1.07

3. Numerical Modeling on Mechanical Properties of the Cement Phosphogypsum Stabilized Soil

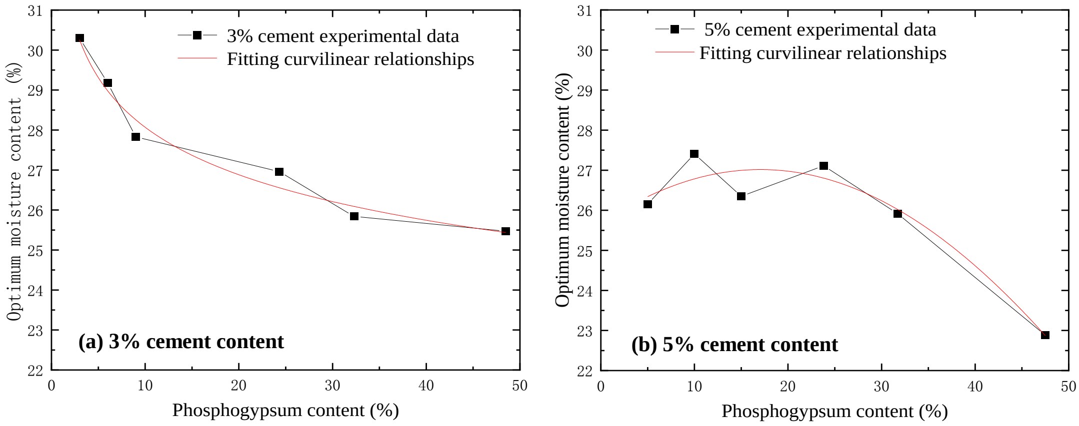
3.1. The Compaction Characteristics

In order to get the theoretical model for describing the compaction properties, liquid-plastic limit properties, the unconfined compressive strength and the fissure rate of cemented phosphogypsum stabilized soil with the changing parameters, due to the existing experimental data have been shown the values of the properties under different parameters, then the published parameters based on the obtained experimental data are chosen in this work for numerical modeling. And indeed, some other parameters also have an effect on the properties of the materials, while in order get the theoretical model, only the published experimental data for the parameters were selected in this work for numerical modeling, and the effects by other parameters including both the experimental work and the theoretical model will be analyzed in a next work. Based on the experimental data in Table 1, the optimal water content was firstly fitted numerically, and the relationship between the optimal water content and phosphogypsum content with different cement contents were given in Figure 2.

Based on the numerical fitting results in Figure 2, the numerical prediction model for the optimal water content H that integrates the cement content, phosphogypsum content and red clay content can be given in Equation 1:

$$H = A_H - B_H m_p - C_H m_c - D_H {m_p}^2 + E_H {m_c}^2 - F_H m_p m_c \tag{1}$$

where H is the optimal moisture content, $A_H = 34.6$, $B_H = 0.078$, $C_H = 2.34$, $D_H = 1.99e-4$, $E_H = 0.21$, $F_H = 0.0014$ are the fitting coefficients, m_p, m_c are the contents of phosphogypsum and cement, respectively, and the $1 - m_c - m_p$ is the content of red clay.



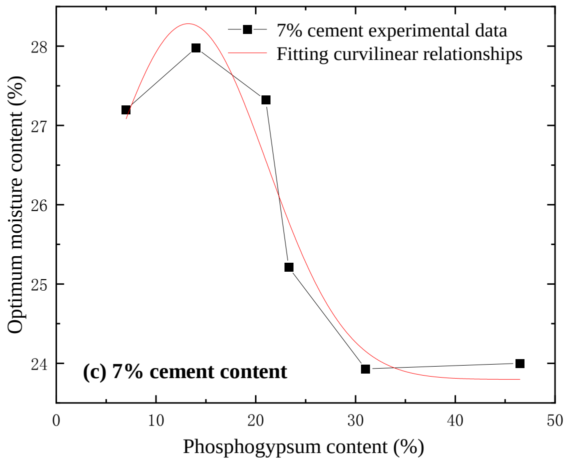


Figure 2. The relationship between the optimal water content and phosphogypsum content with different cement contents

Similarly, the maximum dry density was fitted numerically and the relationship between the maximum dry density and the phosphogypsum content with different cement contents were given in Figure 3.

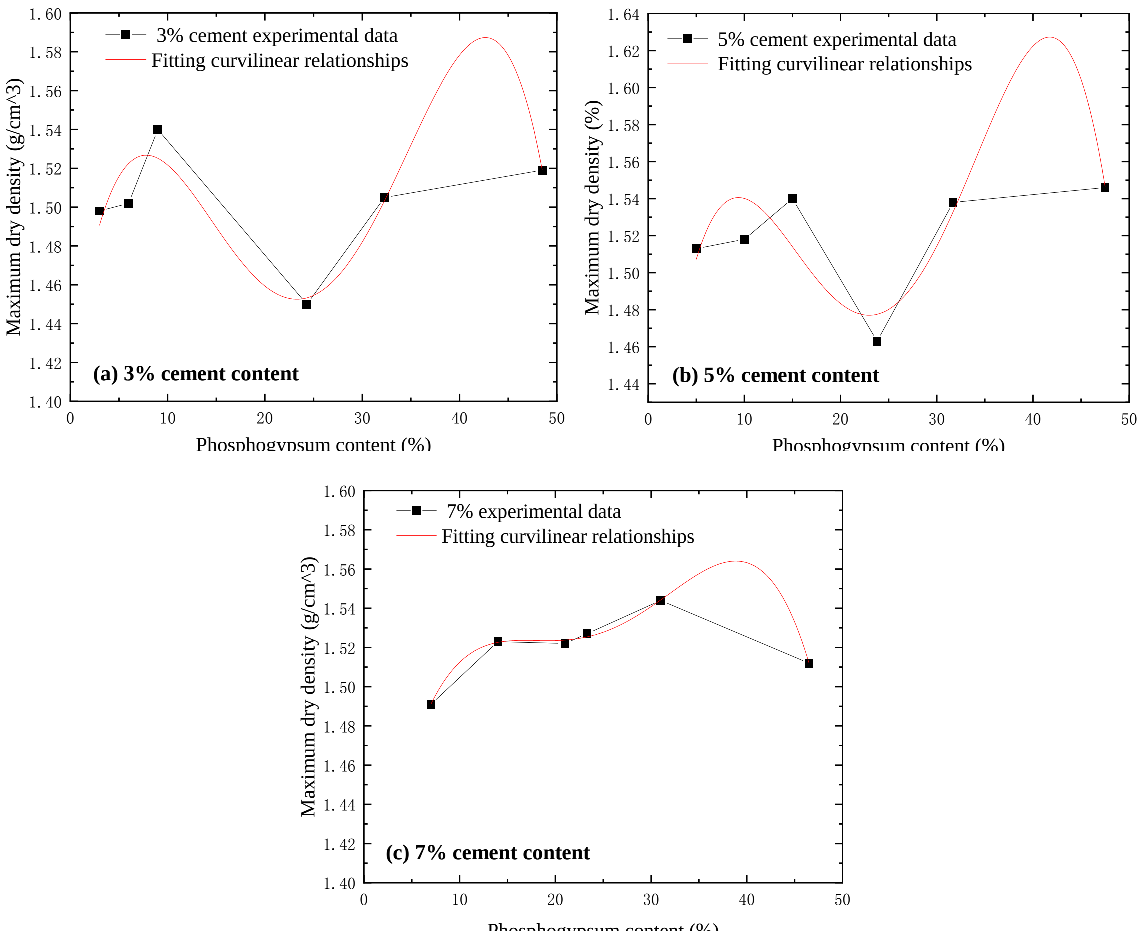


Figure 3. The relationship between the Maximum dry density and phosphogypsum content with different cement contents

Based on the numerical fitting results in Figure 3, the numerical prediction model for the maximum dry density D that integrates the cement content, phosphogypsum content and red clay content can be given in Equation 2:

$$D = A_D + B_D m_p m_c + C_D m_c {m_p}^2 + D_D m_c {m_p}^3 + E_D m_c {m_p}^4 \quad (2)$$

where D is the maximum dry density, $A_D = 1.49, B_D = 0.0012, C_D = -1.04e-4, D_D = 3.35e-6, E_D = -3.4489e-8$ are the fitting coefficients, m_p, m_c are the contents of phosphogypsum and cement, respectively, and the $1 - m_c - m_p$ is the content of red clay.

3.2. The Plastic-Liquid Limit Properties

Based on the experimental data in Table 2, the plastic limit was firstly fitted numerically, and the relationship between the plastic limit and phosphogypsum content with different cement contents were given in Figure 4. Based on the numerical fitting results in Figure 4, the numerical prediction model for the plastic limit Pthat integrates the cement content and phosphogypsum content can be given in Equation 3:

$$P = A_P + B_P m_c + C_P m_p + D_P {m_c}^2 + E_P {m_p}^2 + F_P m_c m_p \tag{3}$$

where P is the plastic limit, $A_P = 51.99$， $B_P = -0.85, C_P = -0.197$, $D_P = 0.146$, $E_P = -0.0035$, $F_P = -0.0195$ are the fitting coefficients, m_p ,m_c are the contents of phosphogypsum and cement, respectively, and the $1 - m_c - m_p$is the content of red clay.

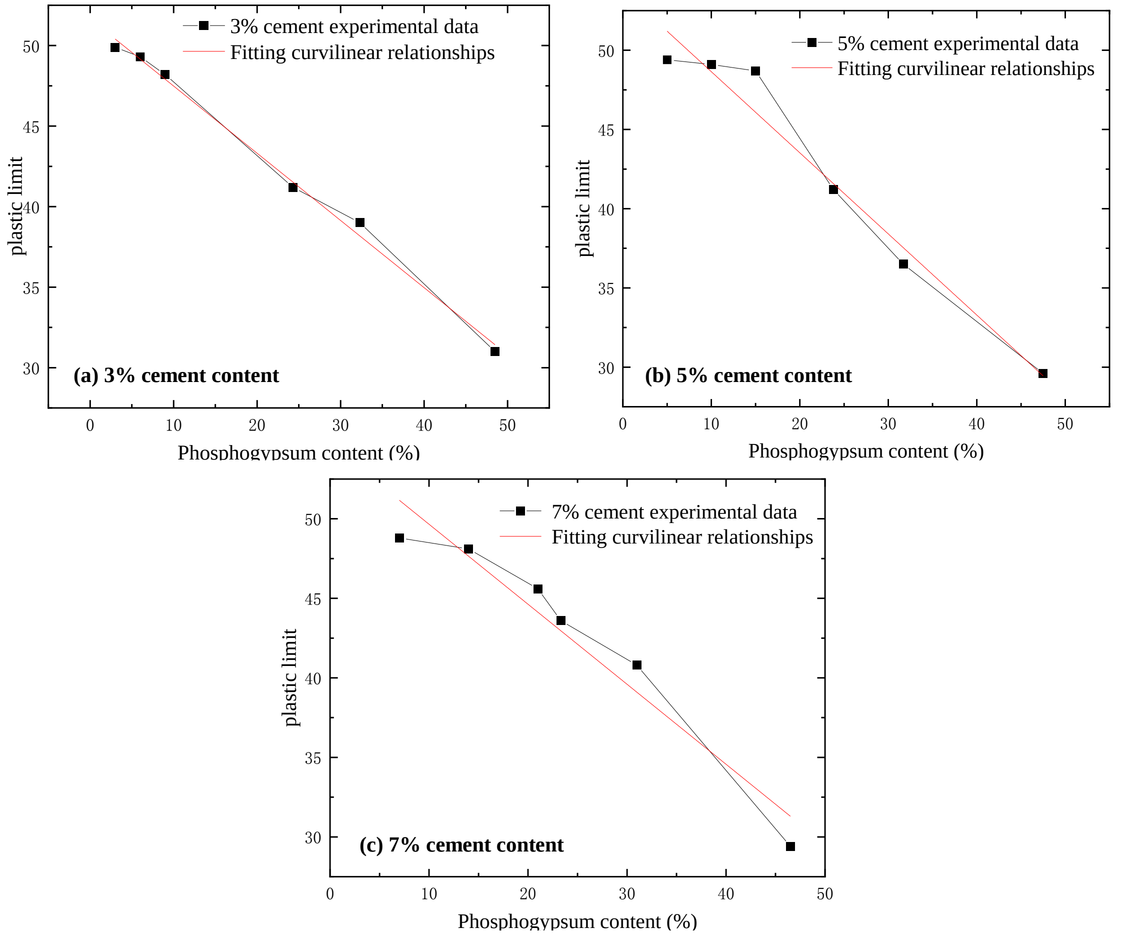
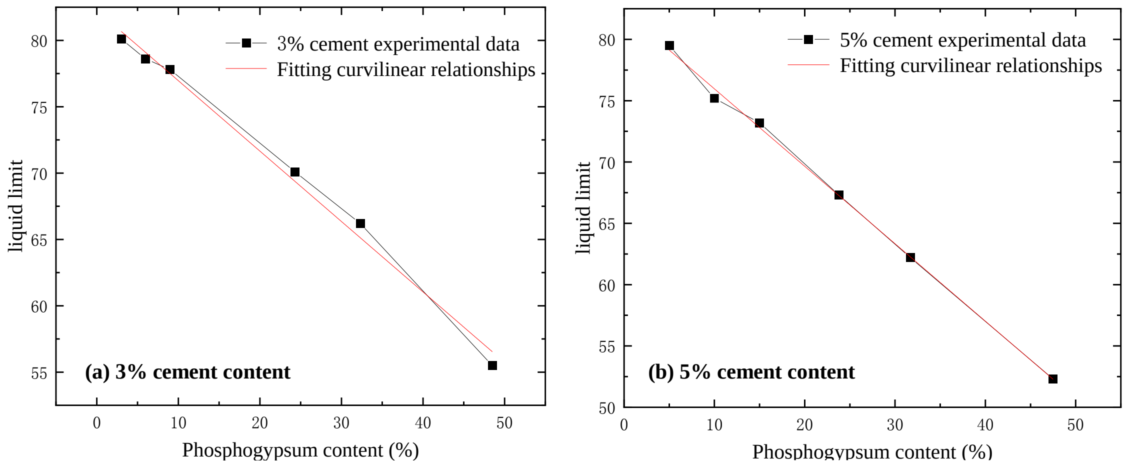


Figure 4. The relationship between the Plastic limit and phosphogypsum content with different cement contents

Similarly, the liquid limit was then fitted numerically, and the relationship between the liquid limit and phosphogypsum content with different cement contents were given in Figure 5.



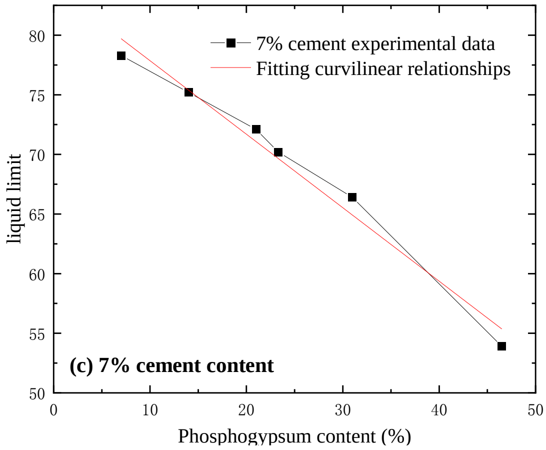


Figure 5. The relationship between the liquid limit and phosphogypsum content with different cement contents

Based on the numerical fitting results in Figure 5, the numerical prediction model for the liquid limit L that integrates the cement content and phosphogypsum content can be given in Equation 4:

$$L = A_L + B_L m_c + C_L m_p + D_L {m_c}^2 + E_L {m_p}^2 + F_L m_c m_p \tag{4}$$

where L is the liquid limit, $A_L = 91.96$, $B_L = -5.12$, $C_L = -0.326$, $D_L = 0.544$, $E_L = -0.0032$, $F_L = -0.020$ are the fitting coefficients, m_p ,m_c are the contents of phosphogypsum and cement, respectively, and the $1 - m_c - m_p$is the content of red clay.

Finally, the plasticity index was fitted numerically and the relationship between the plasticity index and the phosphogypsum content with different cement contents were given in Figure 6. Based on the numerical fitting results in Figure 6, the numerical prediction model for the plasticity index Y that integrates the cement content, phosphogypsum content and red clay content can be given in Equation 5:

$$Y = A_Y + B_Y m_c + C_Y m_c m_p + D_Y m_c {m_p}^2 + E_Y m_c {m_p}^3 + F_Y m_c {m_p}^4 \tag{5}$$

where Y is the plasticity index, $A_Y = 27.79$, $B_Y = 1.51$, $C_Y = -0.268$, $D_Y = 0.014$,$E_Y = -3.01e - 4$, $F_Y = 2.14e - 6$ are the fitting coefficients, m_p, m_c are the contents of phosphogypsum and cement, respectively, and the $1 - m_c - m_p$ is the content of red clay.

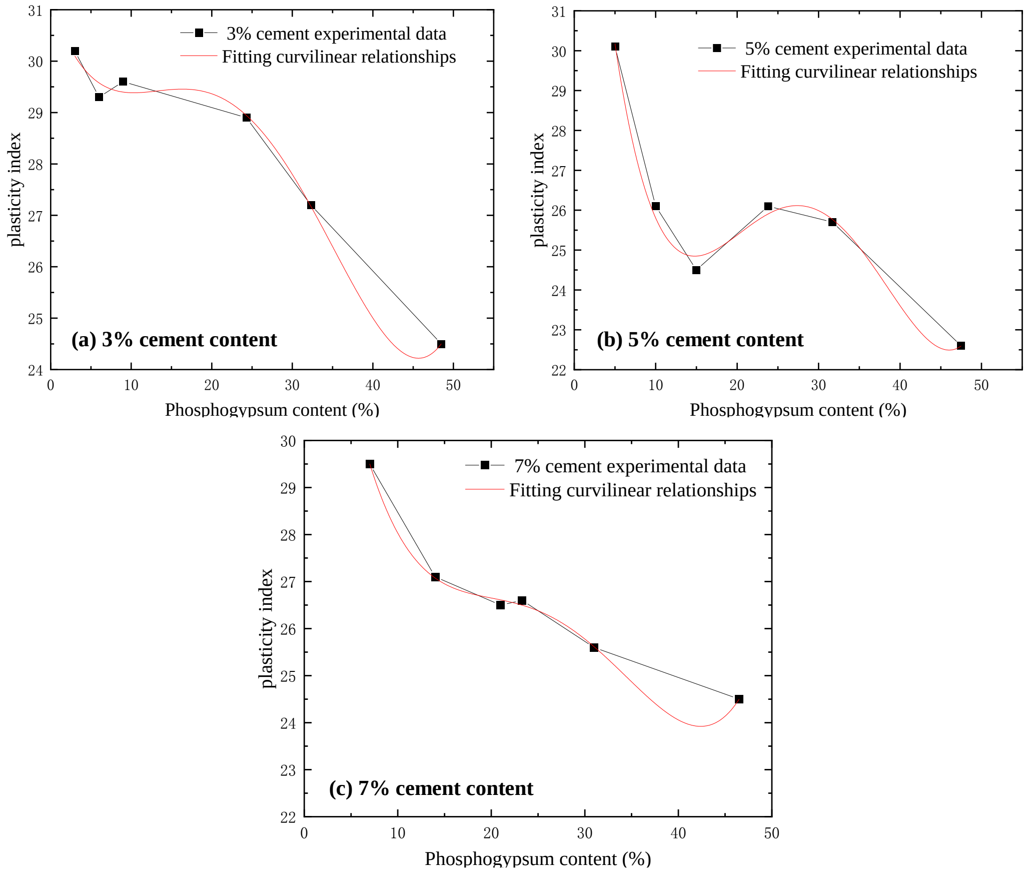


Figure 6. The relationship between the plasticity index and phosphogypsum content with different cement contents

3.3. The Unconfined Compressive Strength

Based on the experimental data in Table 3, the unconfined compressive strength was fitted numerically. The relationship between the unconfined compressive strength and phosphogypsum content with different cement contents were given in Figure 7, and the relationship between the unconfined compressive strength and phosphogypsum content with different curing times were given in Figure 8. Then based on the numerical fitting results in Figures 7 and 8, the numerical prediction model for the unconfined compressive strength U that integrates the cement content, phosphogypsum content, red clay content and the curing times can be given in Equation 6:

$$U = A_U {m_p}^2 t^2 m_c + B_U {m_p}^2 t m_c + C_U {m_p}^2 m_c + D_U m_p t^2 m_c + E_U m_p t m_c + F_U m_p m_c + G_U t^2 m_c + H_U t m_c + I_U m_c + J_U \tag{6}$$

where U is the unconfined compressive strength, t is the curing time, m_p ,m_c is the content of phosphogypsum and cement respectively, and $1 - m_c - m_p$ is the content of red clay, $A_U = 3.6e-7$, $B_U = -1.47e-5$, $C_U = 2.73e-4$, $D_U = -2.39e-6$, $E_U = 8.3e-5$, $F_U = -0.00295$, $G_U = -4.29e-4$, $H_U = 0.02018$, $I_U = -0.118$ and $J_U = 1.037$ are the fitting coefficient.

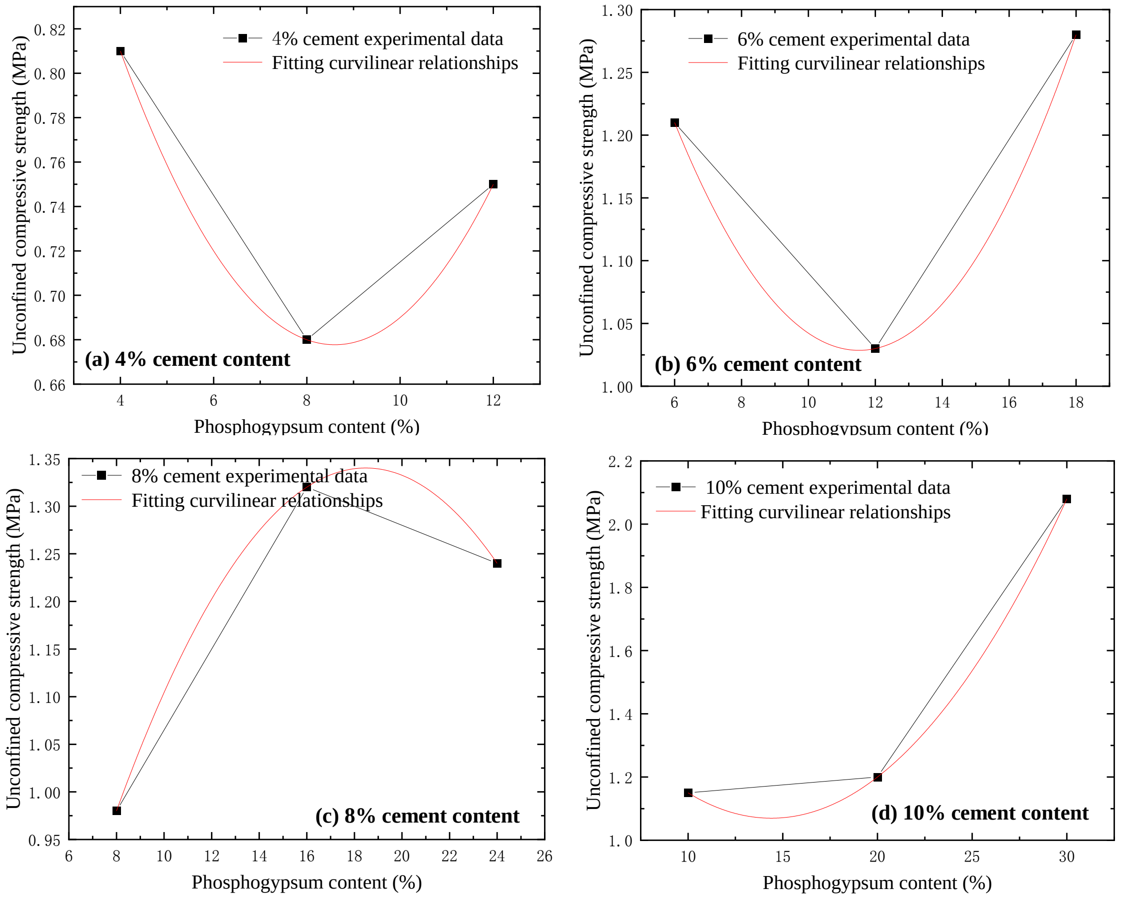
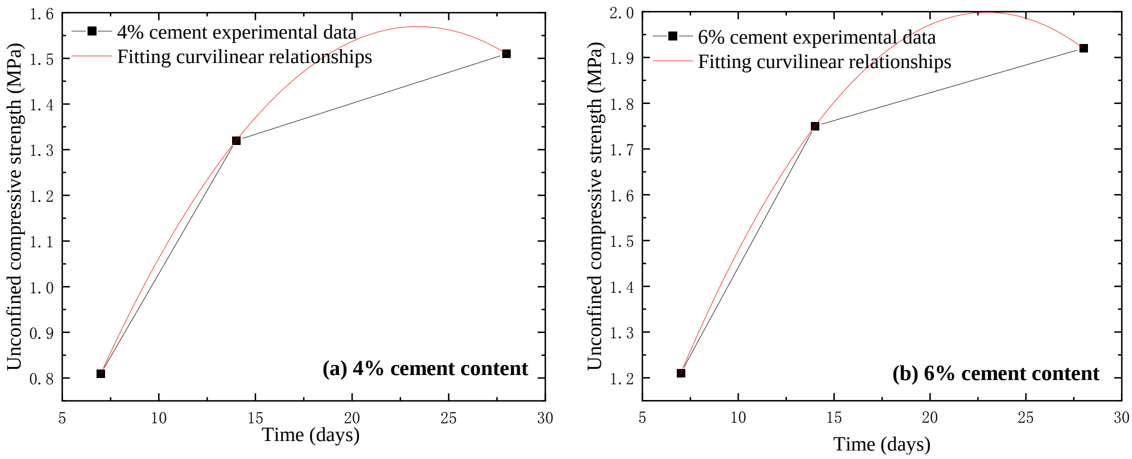


Figure 7. The relationship between the unconfined compressive strength and phosphogypsum content with different cement contents for the curing time is 7 days



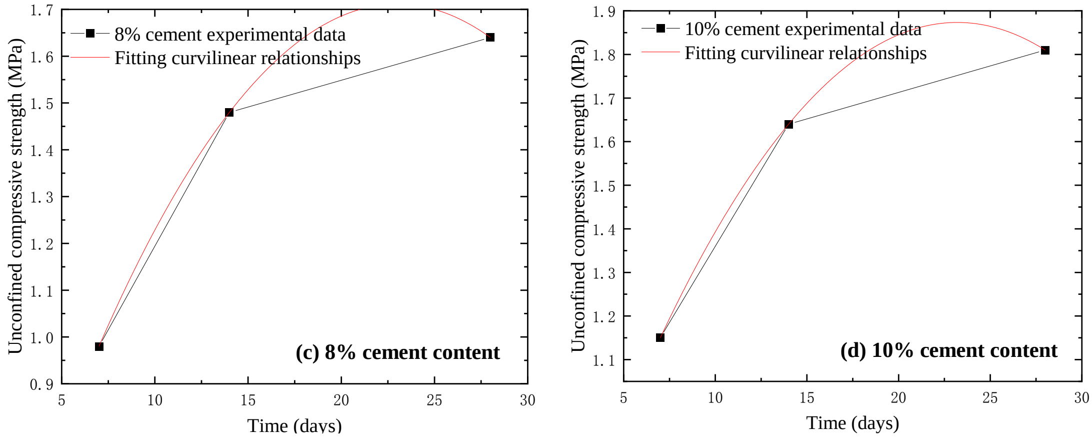


Figure 8. The relationship between the unconfined compressive strength and curing times with different cement contents for the phosphogypsum content is same as the cement content

3.4. The Fissure Rate

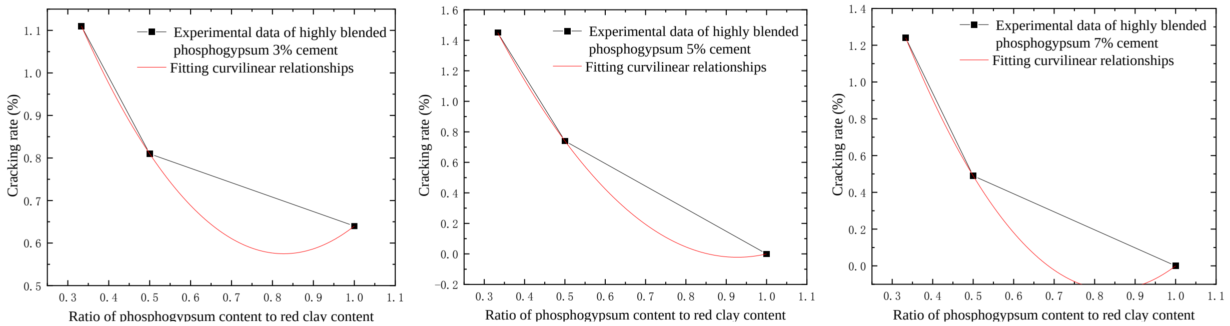
Based on the experimental data in Tables 4 and 5, the fissure rate for high-doped phosphogypsum and the fissure rate for low-doped phosphogypsum were fitted numerically, respectively. The relationship between the fissure rate for high-doped phosphogypsum and phosphogypsum content with different cement contents was given in Figure 9, and the relationship between the fissure rate for low-doped phosphogypsum and phosphogypsum content with different cement contents was given in Figure 10. Then based on the numerical fitting results in Figures 9 and 10, the numerical prediction model for fissure rate for both the high-doped phosphogypsum and the low-doped phosphogypsum that integrates the cement content, phosphogypsum content, and the red clay content can be given in Equations 7 and 8.

$$F_{high} = A_F m_c + B_F m_c x_{high} + C_F m_c {x_{high}}^2 + D_F \quad (7)$$

where F_{high} is the fissure rate of highly doped phosphogypsum, m_p, m_c are the contents of phosphogypsum and cement, respectively, $A_F = 0.402$, $B_F = -1.35$, $C_F = 0.776$ and $D_F = 1.066$ is the fitting coefficient, x_{high} is the ratio of phosphogypsum content to red clay content.

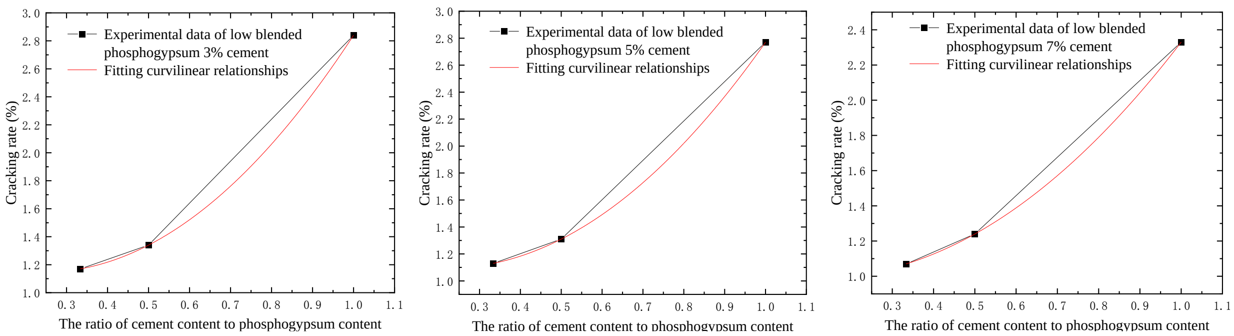
$$F_{low} = A_{F1} m_c + B_{F1} m_c x_{low} + C_{F1} m_c {x_{low}}^2 + D_{F1} \quad (8)$$

where F_{low} is the fissure rate of low doped phosphogypsum, $A_{F1} = -0.15$, $B_{F1} = -0.162$, $C_{F1} = 0.42$, $D_{F1} = 1.985$ is the fitting coefficient, and m_c is the cement content, x_{low} is the ratio of cement content to phosphogypsum content.



(a) 3% cement content **(b) 5% cement content** **(c) 7% cement content**

Figure 9. The relationship between the fissure rate of high doped phosphogypsum and the phosphogypsum with different cement contents



(a) 3% cement content **(b) 5% cement content** **(c) 7% cement content**

Figure 10. The relationship between the fissure rate of low doped phosphogypsum and the phosphogypsum with different cement contents

4. Validation of the Model

4.1. The Compaction Characteristics

In order to compare the model prediction results with the experimental data, some assumptions on the values of some fixed variables were made during the analytical processes. In order to verify the correctness of the numerical model, several groups of specific data for the fixed variates were assumed as examples for the numerical calculation, and then the calculation results of the model can be compared with the published experimental data by using the assumption values of the fixed variates and the numerical model.

The optimum water content of cement phosphogypsum-stabilized soil with different cement contents, different phosphogypsum contents, and different red clay contents can be predicted by Equation 1. The comparison of the predicted data with the experimental data under the selected special cement content, phosphogypsum content, and red clay content is shown in Table 6, from which it can be concluded that the difference between the model predicted results and the experimental data is small, which proves the correctness of the predicted model in this paper.

Similarly, Equation 2 can be used to predict the maximum dry density of cemented phosphogypsum-stabilized soil with different cement contents, different phosphogypsum contents, and different red clay contents. The comparison of the predicted data with the experimental test data under the selected special cement content, phosphogypsum content, and red clay content is shown in Table 7, from which it can be concluded that the difference between the model-predicted results and the experimental data is small, which proves the correctness of the model in this paper.

Table 6. Validation of the model for the optimum water content

Special point selection	Results	Formula results	Error
m_c =3, m_p =3	30.3	29.36	3%
m_c=5, m_p =23.8	24	25.87	7.7%
m_c=7, m_p =46.5	27.11	24.04	9%

Table 7. Validation of the model for the maximum dry density

Special point selection	Results	Formula results	Error
m_c=3, m_p=3	1.498	1.498	0%
m_c=5, m_p=23.8	1.463	1.50	2.5%
m_c=7, m_p=46.5	1.512	1.535	1.5%

4.2. The Plastic-liquid Limit Properties

The plastic limit of cemented phosphogypsum-stabilized soil under different cement content, different phosphogypsum content, and different red clay content can be predicted by Equation 3. The comparison of the predicted data with the experimental data under the selected special cement content, phosphogypsum content, and red clay content is shown in Table 8, from which it can be concluded that the difference between the predicted results and the experimental data is small, which proves the correctness of the present model.

Table 8. Validation of the model for the plastic limit

Special point selection	Results	Formula results	Error
m_c=3, m_p=3	49.9	49.95	0.1%
m_c=5, m_p=23.8	41.2	42.19	2.4%
m_c=7, m_p=46.5	29.4	30.09	2.3%

The liquid limit of cement-phosphogypsum-stabilized soil can be predicted by Equation 4 for different cement contents, different phosphogypsum contents, and different red clay contents. The comparison of the predicted data with the experimental data under the same selected special cement content, phosphogypsum content, and red clay content is shown in Table 9, from which it can be concluded that the results predicted by the model show a low difference from the experimental test data, which proves the correctness of the numerical model.

Table 9. Validation of the model for liquid limit

Special point selection	Results	Formula results	Error
m_c=3, m_p=3	80.1	80.341	0.3%
m_c=5, m_p=23.8	67.3	68.003	1%
m_c=7, m_p=46.5	53.9	54.157	0.46%

The plasticity index of cemented phosphogypsum-stabilized soil with different cement content, different phosphogypsum content, and different red clay content can be predicted by Equation 5. The comparison of the predicted data with the experimental data under the selected special cement content, phosphogypsum content, and red clay content is shown in Table 10, from which it can be concluded that the difference between the model-predicted results and the experimental data is small, which proves the correctness of the present numerical model.

Table 10. Validation of plasticity index data fit

Special point selection	Results	Formula results	Error
m_c=3, m_p=3	30.2	30.26	0.19%
m_c=5, m_p=23.8	26.1	26.55	1.7%
m_c=7, m_p=46.5	24.5	23.82	2.7%

4.3. The Unconfined Compressive Strength

Equation 6 can be used to predict the unconfined compressive strength of cemented phosphogypsum-stabilized soil at different times, cement contents, and phosphogypsum contents. The results of the comparison between the predicted work and the experimental data under the selected special time, cement content, and phosphogypsum content are shown in Table 11, from which it can be concluded that the difference between the predicted values and the experimental data is small, which proves the correctness of the present numerical model.

Table 11. Validation of model for the unconfined compressive strength

Special point selection	Results	Formula results	Error
m_c=4, m_p=4,t =7	0.81	1.01	24%
m_c=6, m_p=12,t =14	1.53	1.49	2.6%
m_c=8, m_p=24,t =28	2.05	2.10	2.4%

4.4. The Fissure Ratio

Firstly, the fissure ratio of cemented phosphogypsum-stabilized soil with different cement content and different phosphogypsum content of high-doped phosphogypsum can be predicted by Equation 7. The comparison results between the predicted values under the selected special cement content and phosphogypsum content and the experimental data are shown in Table 12, from which it can be concluded that the difference between the predicted values and the experimental data is small, which proves the correctness of the present numerical model.

Table 12. Validation of model for the fissure ratio under high doped phosphogypsum

Special point selection	Results	Formula results	Error
m_c=3, x_{high} =1	0.64	0.56	12.5%
m_c=5, x_{high} =0.5	0.74	0.68	8.1%
m_c=7, x_{high}=0.333333	1.24	1.34	8%

Equation 8 can be used to predict the fissure ratio of cemented phosphogypsum-stabilized soil with different cement contents and different phosphogypsum contents for the low-doped case. The comparison results of the predicted values under the selected special cement content and phosphogypsum content with the experimental data are shown in Table 13, from which it can be concluded that the difference between the predicted values and the experimental data is small, which proves the correctness of the present numerical model.

Table 13. Validation of the model for the fissure ratio under low doped phosphogypsum

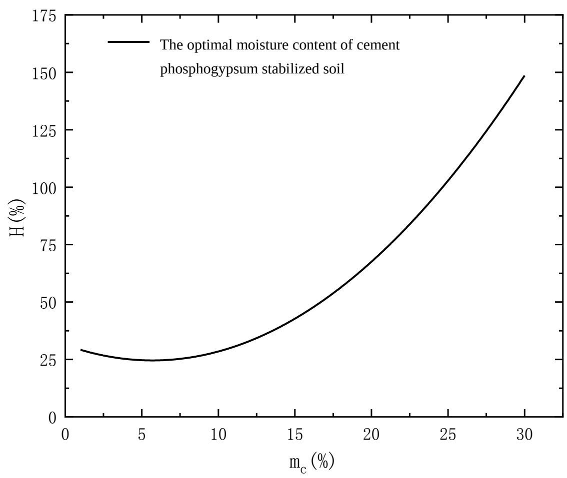
Special point selection	Results	Formula results	Model error
m_c=3, x_{low}=1	2.84	2.36	16.9%
m_c=5, x_{low}=0.5	1.31	1.35	3%
m_c=7, x_{low}=0.333333	1.07	0.88	17%

5. Numerical Prediction of Mechanical Properties of Cemented Phosphogypsum Stabilized Soil

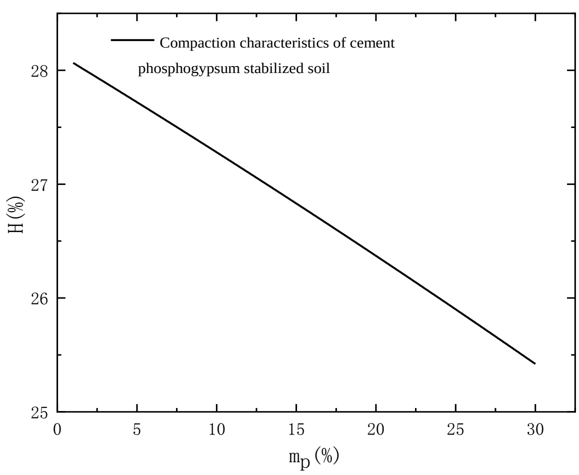
5.1. The Compaction Characteristics

In order to discuss the relationship between the mechanical properties and the different parameters, the fixed variable method was chosen, and some fixed variables are first assumed to be certain values, and then the relationship between the mechanical properties of materials and the changing parameters will be then discussed by using the assumed values of the fixed variables and the numerical model.

Based on Equation 1, the predicted values for the optimal water content with the changing contents of phosphogypsum and cement are given in Figure 11. As an example, for the case with the phosphogypsum content is 37.5%, the relationship between the optimal water content and the changing cement contents can be given in Figure 11(a). As shown in Figure 11(a), when the cement content is low, the optimal moisture content decreases with the increase of cement contents, and when the cement content is larger than 6%, the optimum water content raises with the increasing of cement content. The possible reasons for this phenomenon are: when the cement content is low, the hydration reaction of cement has not been fully activated, but as the amount of cement increases, its hydration products (e.g., calcium hydroxide) begin to increase, and these products help to fill the pores between soil particles and enhance the bonding force between soil particles, thus reducing the moisture content required to achieve the optimum compaction effect. When the cement dosage increases to a certain level, the excess cement may lead to the formation of excessive hydration products and cementitious substances inside the soil, which may shrink during drying and form micro-cracks, and instead, more moisture is needed to fill these newly formed pores to maintain the stability and densification of the soil.



(a) Optimum moisture content vs. cement content



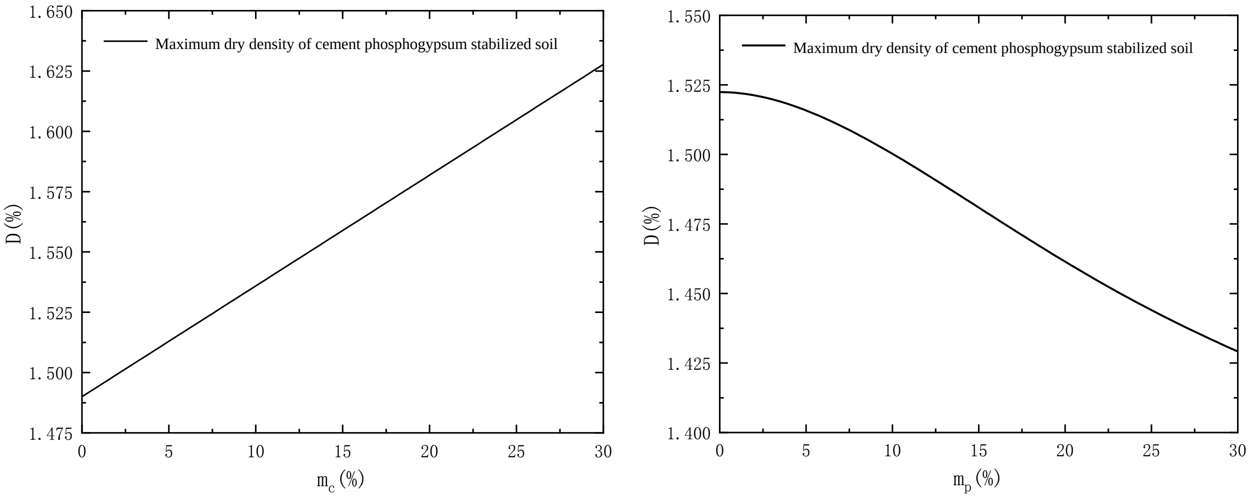
(b) Optimum moisture content vs. phosphogypsum content

Figure 11. Predicted values for the optimal water content with the changing contents of phosphogypsum and cement

Similarly, for the case with the cement content is 5%, the relationship between the optimal water content H and the changing phosphogypsum contents m_p can be given in Figure 11(b). As shown in Figure 11(b), the optimum moisture content of the stabilized soil shows a continuous decrease with the increasing of phosphogypsum content. The possible reasons for this phenomenon are: the fine particles of phosphogypsum can effectively fill the voids between soil particles and reduce the total porosity of the soil, and the certain components in phosphogypsum may react chemically with the hydration products of cement, further promoting the densification and strength enhancement of the soil body. These effects result in a gradual decrease in the water content required to achieve optimum compaction with increasing phosphogypsum content at the same cement admixture.

Based on the Equation 2, the predicted values for the maximum dry density D with the changing contents of phosphogypsum and cement are given in Figure 12. As an example, for the case with the phosphogypsum content is 9%, the relationship between the maximum dry density D and the changing cement contents m_c can be given in Figure 12(a). As shown in Figure 12(a), the maximum dry density raises with the increasing of cement content. The possible reasons for this phenomenon are: the addition of cement can more fully fill the space between phosphogypsum particles, to form a more dense structure.

Similarly, for the case with the cement content is 5%, the relationship between the maximum dry density D and the changing phosphogypsum contents m_p can be given in Figure 12(b). As shown in Figure 12(b), the maximum dry density of the stabilized soil shows a continuous decrease with the increasing of phosphogypsum content. The possible reasons for this phenomenon are: the overmuch phosphogypsum particles can occupy the position of cement particles, reduces the effective cementing area of cement, leads to the decline of the overall structure compactness.



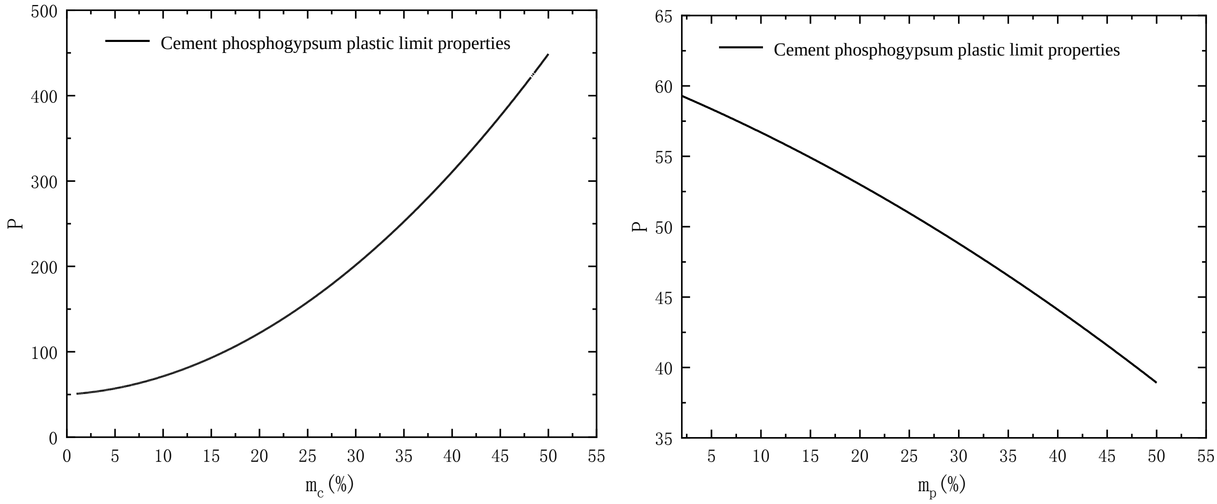
(a) Maximum dry density vs. cement content **(b) Maximum dry density vs. phosphogypsum content**

Figure 12. Predicted values for the maximum dry density with the changing contents of phosphogypsum and cement

5.2. The Plastic-liquid Limit Properties

Based on the Equation 3, the predicted values for the plastic limit $P$with the changing contents of phosphogypsum and cement are given in Figure 13. As an example, for the case with the phosphogypsum content is 9%, the relationship between the plastic limit P and the changing cement contents $m_c$can be given in Figure 13(a). As shown in Figure 13(a), the plastic limit raises with the increasing of cement content. The possible reasons for this phenomenon are: with the increasing cement contents, more complex gel networks and skeleton structures begin to form within the soil. These structures not only enhance the mechanical strength of the soil but also limit the free movement of soil particles, thus increasing the plasticity of the soil. In addition, excess cement may lead to the production of excessive hydration products within the soil, and these products may generate shrinkage stresses during drying, further affecting the plastic behavior of the soil. Therefore, the plasticity limit raises with the increasing of cement content at this stage.

Similarly, for the case with the cement content is 5%, the relationship between the plastic limit $P$and the changing phosphogypsum contents $m_p$can be given in Figure 13(b). As shown in Figure 13(b), the plastic limit of the stabilized soil shows a continuous decrease with the increasing of phosphogypsum content. The possible reasons for this phenomenon are: due to the fine particles of phosphogypsum, the tiny pores between soil particles can be effectively filled and the total porosity of the soil were then reduced, which can make the contact between soil particles closer, thus reducing the plasticity and mobility of the soil body. Besides, the certain components in phosphogypsum may react chemically with cement hydration and may product to new mineral phases, which may have an effect on the microstructure and mechanical properties of the soil. furthermore, the addition of phosphogypsum may also change the charge distribution and wettability of the surface of soil particles, which can also affect its plastic behavior.

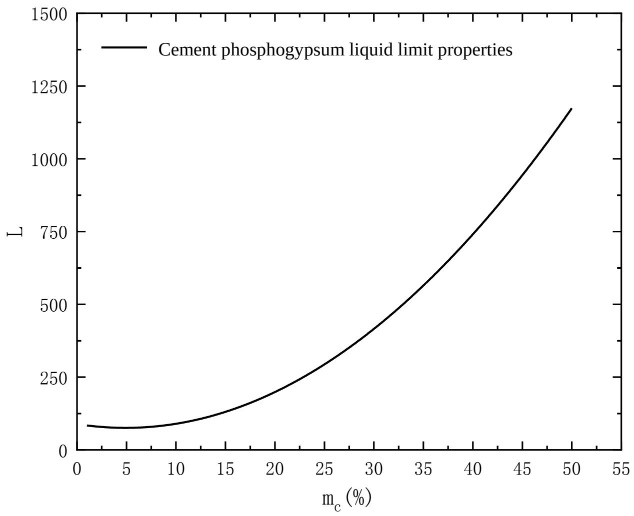


(a) Plastic limit vs. cement content **(b) Plastic limit vs. phosphogypsum content**

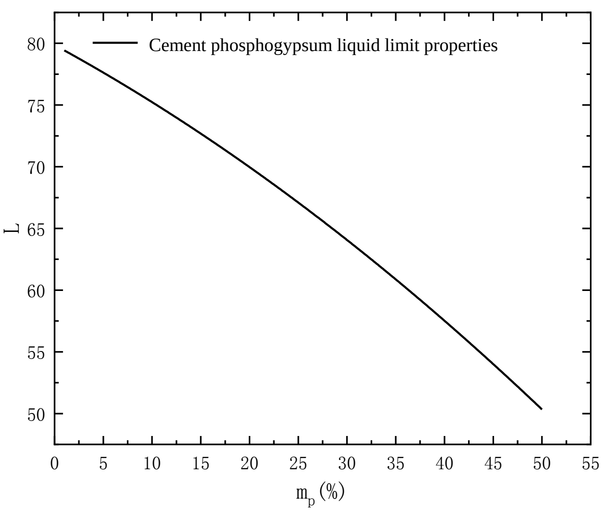
Figure 13. Predicted values for the plastic limit with the changing contents of phosphogypsum and cement

Based on the Equation 4, the predicted values for the liquid limit L with the changing contents of phosphogypsum and cement are given in Figure 14. As an example, for the case with the phosphogypsum content is 9%, the relationship between the liquid limit L and the changing cement contents m_c can be given in Figure 14(a). As shown in Figure 14(a), the liquid limit raises with the increasing of cement content. The possible reasons for this phenomenon are: due to the addition of cement, hydration reaction will happen, and form the gelling products, which can fill the space between the soil particles and make the soil structure denser. Such hydration reaction requires a certain amount of water, and the liquid limit then increases with the increasing of cement content. .

Similarly, for the case with the cement content is 5%, the relationship between the liquid limit $P$and the changing phosphogypsum contents $m_p$can be given in Figure 14(b). As shown in Figure 14(b), the liquid limit of the stabilized soil shows a continuous decrease with the increasing of phosphogypsum content. The possible reasons for this phenomenon are: on the one hand, the phosphogypsum can be treated as a fine-grained material to fill the pores between soil particles and then the contents of free water were reduced. Besides, the phosphogypsum may react chemically with the hydration products of the cement to further enhance the bonding between soil particles.



(a) Liquid limit vs. cement content

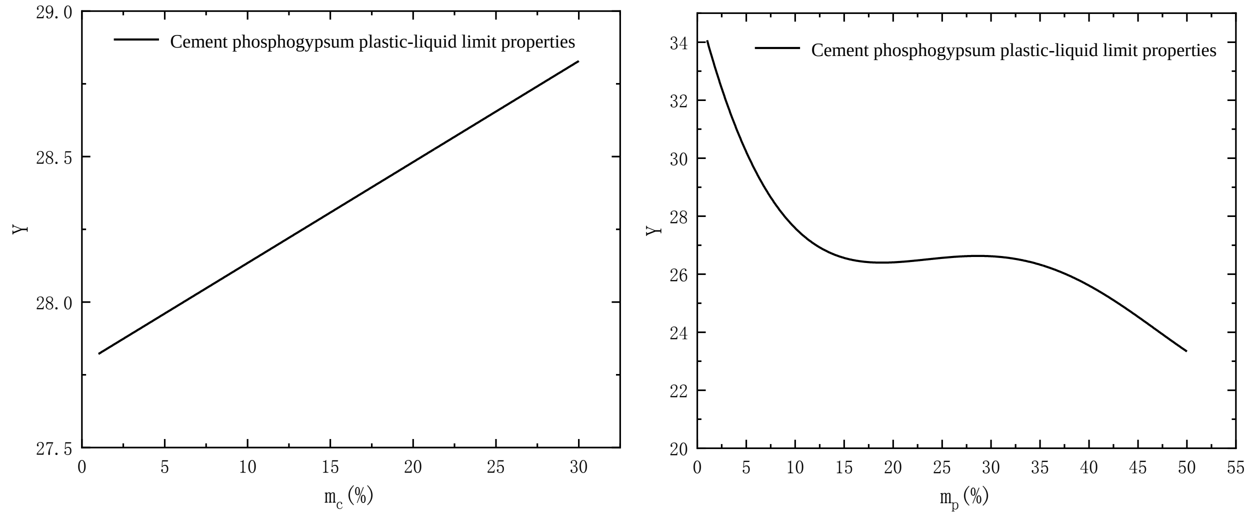


(b) Liquid limit vs. phosphogypsum content

Figure 14. Predicted values for the liquid limit with the changing contents of phosphogypsum and cement

Based on the Equation 5, the predicted values for the plasticity index $Y$with the changing contents of phosphogypsum and cement are given in Figure 15. As an example, for the case with the phosphogypsum content is 9%, the relationship between the plasticity index Y and the changing cement contents $m_c$can be given in Figure 15(a). As shown in Figure 15(a), the plasticity index raises with the increasing of cement content. The possible reasons for this phenomenon are: the incorporation of cement significantly improved the interaction between soil particles and enhanced the structural stability and plasticity of the soil. The hydration products of cement formed a strong bonding network between the soil particles, which enabled the soil to maintain a plastic state in a wider range of water content, thus improving the construction performance and long-term stability of the soil.

Similarly, for the case with the cement content is 5%, the relationship between the plasticity index $Y$and the changing phosphogypsum contents $m_p$can be given in Figure 15(b). As shown in Figure 15(b), the plasticity index of the stabilized soil shows a firstly decreasing, then raising, and finally decreasing with the increasing of phosphogypsum contents. The possible reasons for this phenomenon are: the addition of phosphogypsum may first fill the tiny pores between soil particles and reduce the free water content, thus decreasing the plasticity of the soil in the initial process. As the phosphogypsum content increases further, its chemical reaction with the hydration products of the cement may begin to dominate, generating a new mineral phase that enhances the bond between the soil particles and allows the plasticity index to rebound in the second process. When the phosphogypsum content continue to be increased, it may adversely affect the soil structure, such as for overfilling of the phosphogypsum, it may lead to the destruction of the soil pore structure or by the formation of chemical reaction products that are not conducive to stabilization, leading to a further decline in the plasticity index.



(a) Plasticity index vs. cement content **(b) Plasticity index vs. phosphogypsum content**

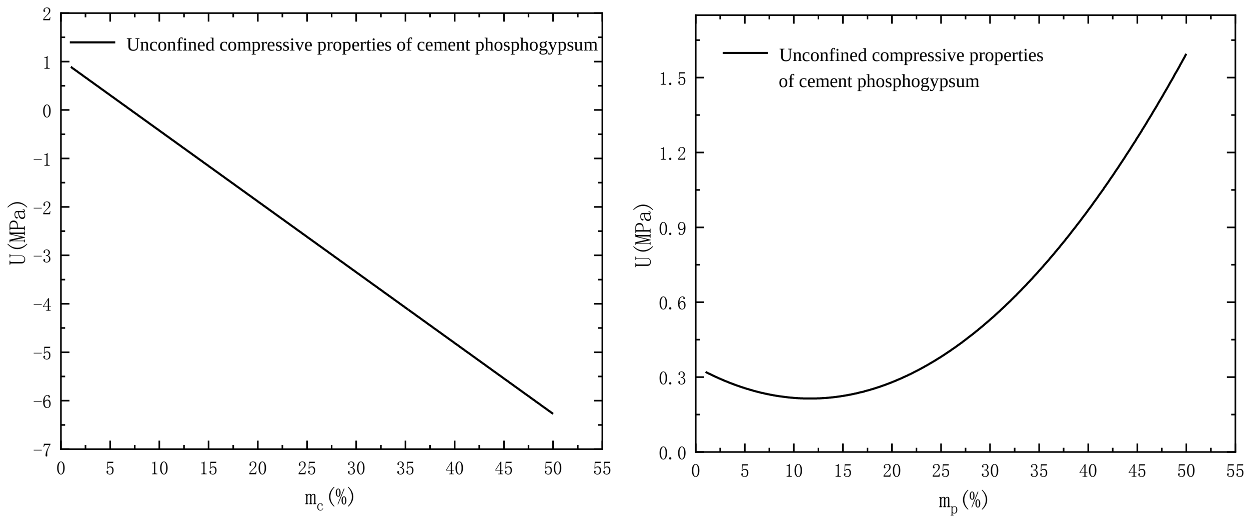
Figure 15. Predicted values for the plasticity index with the changing contents of phosphogypsum and cement

5.3. The Unconfined Compressive Strength Properties

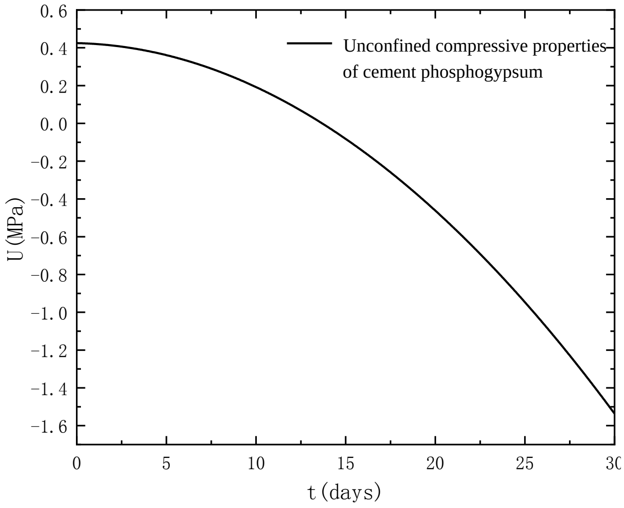
Based on the Equation 6, the predicted values for the unconfined compressive strength U with the changing of the phosphogypsum and cement contents and the curing times t are given in Figure 16. As an example, for the case with the phosphogypsum content is 9% and the curing times is 7 days, the relationship between the unconfined compressive strength $U$and the changing cement contents $m_c$can be given in Figure 16(a). As shown in Figure 16(a), the unconfined compressive strength decreases with the increasing of cement contents. The possible reasons for this phenomenon may be due to the complex chemical reaction between cement and phosphogypsum and red clay, and the high cement content may have led to excessive hydration or unfavorable changes in the pore structure, thus reducing the overall strength.

Similarly, for the case with the cement content is 5% and the curing times is 7 days, the relationship between the unconfined compressive strength $U$and the changing phosphogypsum contents $m_p$can be given in Figure 16(b). As shown in Figure 16(b), the unconfined compressive strength of the stabilized soil shows a firstly decreasing and then raising with the increasing of phosphogypsum contents. The possible reasons for this phenomenon are: an excessive amount of phosphogypsum may react unfavorably with the hydration products of the cement or affect the pore distribution, which may reduce the strength. However, with further increases in phosphogypsum content, new mineral phases or structures favorable for strength development may be formed, leading to strength recovery.

Also, for the case with the cement content is 5% and the phosphogypsum content is phosphogypsum content is 9%, the relationship between the unconfined compressive strength $U$and the changing curing times t can be given in Figure 16(c). As shown in Figure 16(c), the unconfined compressive strength of the stabilized soil shows decrease with the increasing of curing times. The possible reasons for this phenomenon are: due to the gradual completion of the cement hydration reaction with times, it is accompanied by the creation and expansion of microcracks, changes in the pore structure, and possibly environmental factors, and the overall strength will be decreased.



(a) Unconfined compressive strength vs. cement contents **(b) Unconfined compressive strength vs. phosphogypsum contents**



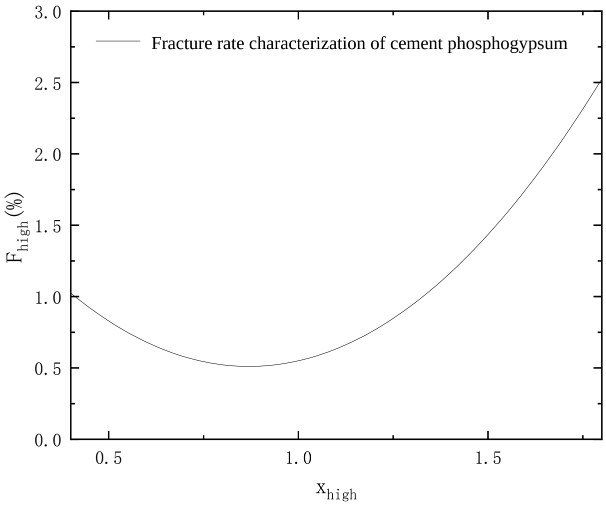
(c) Unconfined compressive strength vs. curing times

Figure 16. Predicted values for the unconfined compressive strength with the changing phosphogypsum and cement contents and the curing times

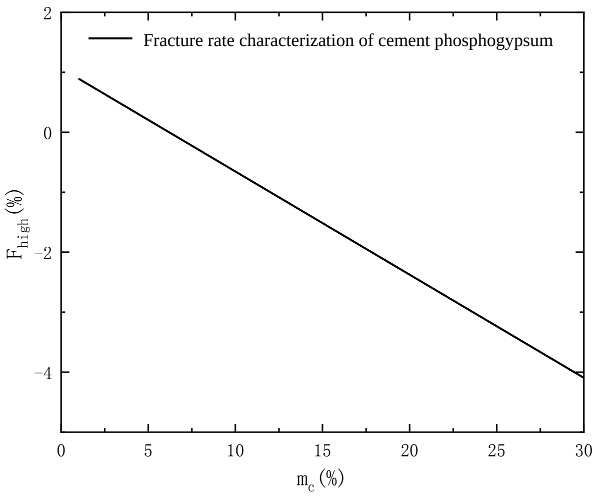
5.4. The Fissure Rate

Based on the Equations 7 and 8, the predicted values for the fissure rate for both the highly doped phosphogypsum F_{high} and the low doped phosphogypsum F_{low} with the changing contents of the phosphogypsum, cement and the red clay are given in Figure 17. As an example, for the case with the cement content is 3%, the relationship between the fissure rate for the highly doped phosphogypsum F_{high} and the changing ratios of the cement content and the phosphogypsum content can be given in Figure 17(a). The ratio of phosphogypsum content to red clay content, can also be defined as "phosphogypsum and red clay ratio". As shown in Figure 17(a), the fissure rate first decreases and then increases with the increasing phosphogypsum and red clay ratios. The possible reasons for this phenomenon are: the moderate addition of phosphogypsum may help to fill the pores and improve the inter-particle interactions, thus reducing the fracture rate. However, when the phosphogypsum and red clay ratio is too high, the phosphogypsum may have detrimental effects, such as reacting with cement hydration products to produce an expansive mineral phase, leading to internal stress concentration and microcrack formation, which in turn increases the fracture rate.

For the case with the phosphogypsum and red clay ratio is 1%, the relationship between the fissure rate for the highly doped phosphogypsum F_{high} and the changing cement contents $m_c$can be given in Figure 17(b). As shown in Figure 17(b), the fissure rate for the highly doped phosphogypsum decreases with the increasing of cement contents. The possible reasons for this phenomenon are: because the hydration products of cement as a stabilizer can significantly enhance the bonding between soil particles and form a denser and more stable structure, thus effectively reducing the fracture rate. In addition, the addition of cement can improve the pore structure and water distribution of the soil, further improving the mechanical properties of material.



(a) Fissure rate vs. phosphogypsum and red clay ratio

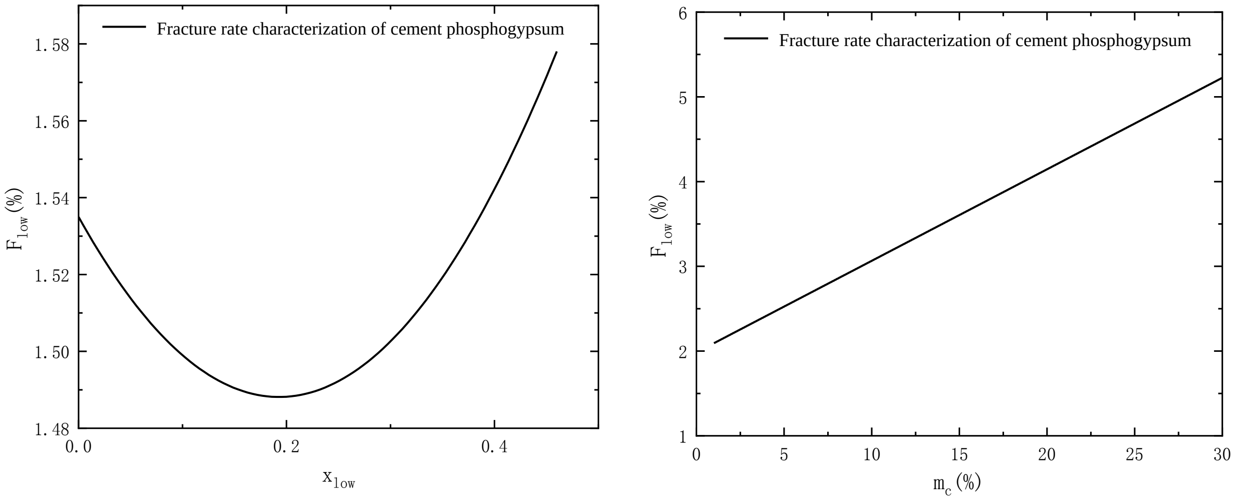


(b) Fissure rate vs. cement content

Figure 17. Predicted values for the fissure rate for high doped phosphogypsum with the changing contents of phosphogypsum, red clay and cement

As another example, for the case with the cement content is 3%, the relationship between the fissure rate for the low doped phosphogypsum F_{low} and the changing ratios of the cement content and the phosphogypsum content can be given in Figure 18(a). As shown in Figure 18(a), the fissure rate for the low doped phosphogypsum firstly decreases and then raises with the increasing of cement contents. The possible reasons for this phenomenon are: in the initial stage, with the increase of phosphogypsum and red clay ratio, the gelation effect of cement is gradually enhanced, due to better filling the void between phosphogypsum particles, and then the fracture rate can be reduced. However, when the phosphogypsum and red clay ratio continue increases, excessive cement may cause stress concentration in the structure, or reduce the uniformity of the overall structure, which can reduce the fracture resistance and lead to an increase in the fracture rate.

For the case with the phosphogypsum and red clay ratio is 1%, the relationship between the fissure rate for low doped phosphogypsum F_{low} and the changing cement contents $m_c$can be given in Figure 18(b). As shown in Figure 18(b), the fissure rate for the low doped phosphogypsum raises with the increasing of cement contents. The possible reasons for this phenomenon are: with the increasing of the cement content, the concentration of internal stresses and the expansion of microcracks will happen, which can increase the overall fracture rate.



(a) Fissure rate vs. phosphogypsum and red clay ratio **(b) Fissure rate vs. cement content**

Figure 18. Predicted values for the fissure rate for low doped phosphogypsum with the changing contents of phosphogypsum, red clay and cement

6. Conclusion

This paper gives a numerical analysis model for simulating the mechanical properties, including the optimum moisture content, maximum dry density, plastic limit, liquid limit, plasticity index, unconfined compressive strength, and fissure rate of cement phosphogypsum-stabilized soil. The main conclusions are as follows. Firstly, the mathematical analysis model given in this paper can effectively predict the mechanical properties of cement phosphogypsum-stabilized soil and can give the relationship between the influences of different influencing factors on the mechanical properties of cement phosphogypsum. Secondly, the results show that with the increasing cement content, the optimum moisture content, maximum dry density, plastic limit, liquid limit, plasticity index, and the fissure rate for high-doped phosphogypsum show a gradual increasing trend, and the unconfined compressive strength and the fissure rate for high-doped phosphogypsum show a decreasing trend. With the increasing phosphogypsum content, the optimum moisture content, maximum dry density, plastic limit, liquid limit, and plasticity index show a gradual decreasing trend, and the unconfined compressive strength and the fissure rate for both the high doped phosphogypsum and the low doped phosphogypsum show an increasing trend. Thirdly, the method and mathematical model proposed in this paper can be used to predict the results with other different phosphogypsum and cement contents, or using the method proposed in this paper to carry on the other relevant research work analysis, and the purpose of reducing the number of tests will be then achieved. At last, the research work in this paper can provide theoretical support for further research and description of the mechanical properties of cement phosphogypsum-stabilized soil.

7. Declarations

7.1. Author Contributions

Conceptualization, W.M. and B.L.; methodology, B.L.; software, W.M.; validation, W.M., B.L., and L.Z.; formal analysis, W.M.; investigation, B.L.; resources, B.L.; data curation, L.Z.; writing—original draft preparation, W.M.; writing—review and editing, W.M.; supervision, B.L.; project administration, L.Z.; funding acquisition, L.Z. All authors have read and agreed to the published version of the manuscript.

7.2. Data Availability Statement

The data presented in this study are available on request from the corresponding author.

7.3. Funding

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7.4. Institutional Review Board Statement

Not applicable.

7.5. Informed Consent Statement

Not applicable.

7.6. Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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